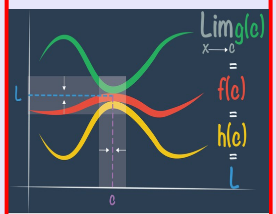
Math 261

Fall 2022

Lecture 50

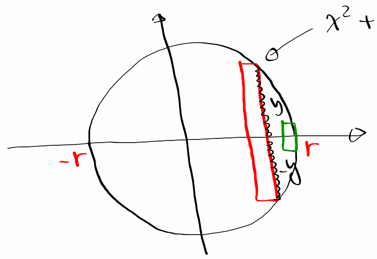


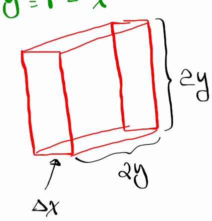
Feb 19-8:47 AM

Consider Solid S with circular base of radius r. Parallel cross-sections perpendicular to the base are squares. Find its volume.

$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$





$V = \int_{-r}^{r} 4y^2 \, dx$

$= \int_{-r}^{r} 4(r^2 - x^2) \, dx$

$V = LWH$

$= 2y \cdot 2y \cdot \Delta x$

$= 4y^2 \Delta x$

$= 2 \cdot 4 \int_0^r (r^2 - x^2) \, dx = 8\left[r^2 x - \frac{x^3}{3} \right]\Big|_0^r$

$= 8\left[r^3 - \frac{r^3}{3} \right] = 8 \cdot \frac{2r^3}{3} = \boxed{\frac{16r^3}{3}}$

Nov 29-8:45 AM

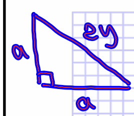
The base of Solid S is a circular base with radius 1 centered at (1,0).

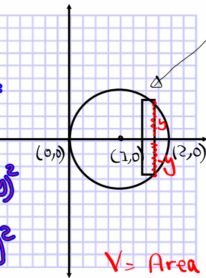
Cross-Sections ⊥ to x-axis are isosceles right triangle with hypotenuse in the base. Find its volume.

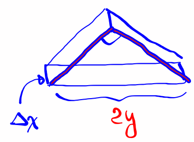
$(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+y^2=1$

$y^2=1-(x-1)^2$







$a^2+a^2=(2y)^2$

$2a^2=4y^2$

$a^2=2y^2$

$a=y\sqrt{2}$

$V=\text{Area of triangle}\cdot\Delta x$

$=\dfrac{a\cdot a}{2}\cdot\Delta x$

$=\dfrac{a^2}{2}\,\Delta x$

$$V=\int_0^2 \frac{2y^2}{2}\,dx=\int_0^2 y^2\,dx=\int_0^2 [1-(x-1)^2]\,dx$$

$$=\int_0^2 (\cancel{1}-x^2+2x-\cancel{1})\,dx=\left(x^2-\frac{x^3}{3}\right)\Big|_0^2=4-\frac{8}{3}=\boxed{\frac{4}{3}}$$

Nov 29-8:54 AM

The base of a Solid is the region bounded by $y=x^2$ and $y=2-x^2$.

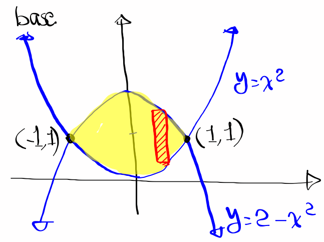
Cross-Sections ⊥ x-axis are squares with one Side on the base

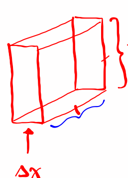
$x^2=2-x^2$

$2x^2=2$

$x^2=1$

$x=\pm1$





Top - Bottom $=2-x^2-x^2=2-2x^2$

$V=LWH$

$=(2-2x^2)\cdot(2-2x^2)\cdot\Delta x$

$$V=\int_{-1}^{1} \underbrace{(2-2x^2)(2-2x^2)}_{\text{even function}}\,dx=2\int_0^1 (4-8x^2+4x^4)\,dx$$

$$=2\left[4x-\frac{8x^3}{3}+\frac{4x^5}{5}\right]\Big|_0^1$$

$$=2\left[4-\frac{8}{3}+\frac{4}{5}-0\right]=\boxed{\frac{64}{15}}$$

Nov 29-9:06 AM

Find f_{ave} for $f(x) = x\sin x^2$ on $[0, 10]$

$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$ when $f(x)$ is cont. on $[a,b]$.

For $f(x) = x\sin(x^2)$

cont. cont. cont.

$f_{ave} = \frac{1}{10-0}\int_0^{10} x\sin x^2\,dx = \frac{1}{10}\int_0^{100}\sin u\,\frac{du}{2}$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x=0 \to u=0$

$x=10 \to u=100$

$f_{ave} = \frac{1}{20}\cdot -\cos u\Big|_0^{100} = \frac{-1}{20}\left[\cos 100 - \cos 0\right]$ ($\cos 0 = 1$)

$= \boxed{\frac{1-\cos 100}{20}}$

Nov 29-9:17 AM

Suppose $f(x)$ is a cont. function

Find $\lim_{h\to 0} f_{ave}$ on the interval $[x, x+h]$.

$f_{ave} = \frac{1}{\cancel{x}+h-\cancel{x}}\int_x^{x+h} f(x)\,dx$

$f_{ave} = \frac{1}{h}\int_x^{x+h} f(x)\,dx$

$f_{ave} = \frac{\int_x^{x+h} f(x)\,dx}{h}$

Now

$\lim_{h\to 0} f_{ave} = \lim_{h\to 0}\frac{\int_x^{x+h} f(x)\,dx}{h} = \boxed{\lim_{h\to 0}\frac{F(x+h)-F(x)}{h}}$

$\int_x^{x+h} f(x)\,dx = F(x)\Big|_x^{x+h} = F(x+h) - F(x)$ $= F'(x)$

$\frac{d}{dx}\left[F(x)\right] = f(x)$ $= \boxed{f(x)}$

$F'(x) = f(x)$

Nov 29-9:23 AM